

Note: Galois theory "in a nutshell"

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A condense vision on Galois group of an equation.
Let be the equation,

$$f(t) = t^4 - 4t^2 - 5 = 0$$

whose roots are, $\alpha, \beta, \gamma, \delta$

Equation coefficients belong to \mathbb{Q}

The enlarged field is $\mathbb{Q}(\alpha, \beta, \gamma, \delta)$

Taken into account the symmetry equations of
the roots (Newton) we write:

$$\alpha^2 + 1 = 0, \alpha + \beta = 0, \delta^2 - 5 = 0, \gamma + \delta = 0, \beta^2 + 1 = 0, \\ \gamma^2 + 1 = 0, \beta\gamma - \alpha\delta = 0, \alpha\gamma - \beta\delta = 0, \gamma^2 - 5 = 0$$

The Galois group is, $G = (I, R, S, T)$

$$R = (\alpha\beta)$$

$$S = (\gamma\delta)$$

$$T = (\alpha\beta)(\gamma\delta)$$

$$I = I$$

$$G \supset (I, R)$$

An equation is solvable when the Galois group can be put
as the sum of a subgroup of G and the multiplication of this subgroup by one
or several automorfisms of G .

The split ought to be normal or proper(Galois).That means that,
 $G = U + S_1U$,and besides, $G = U + US_1$ In our case, given G , U is the subgroup
and T is the automorfism.

$$Si, U = (I, R) \text{ y } S_1 = T = (\alpha\beta)(\gamma\delta)$$

We will write,

$$G = U + S_1U$$

and

$$G = U + US_1$$

That completes the example.